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FOSTERING CONJECTURE CAPACITY AND SOUND REASONING FOR STUDENT IN GEOMETRY TEACHING AT HIGH SCHOOL

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LIST OF WORKS PUBLISHED BY THE AUTHOR RELATED TO TOPIC OF THE DISSERTATION

I. Published journals

1. Vu Dinh Chinh (2013), *Fostering conjecture capacity for students at Grade 11 by using analogy in teaching spatial geometry*, Journal of Science No. 58, pp. 49-56, Hanoi National University of Education.
2. Vu Dinh Chinh (2014), *Training conjecture activities for high school students by using dialectical relationship between the categories of general and particular in teaching spatial geometry*, Journal of Science No. 59, pp. 200-209, Hanoi National University of Education.
3. Vu Dinh Chinh (2015), *Training conjecture activities for students by using inductive reasoning in teaching Grade 10th geometry*, Journal of Science Education No. 119, August 2015, pp. 42-44 and pp. 48, Hanoi.
4. Vu Dinh Chinh (2015), *Training conjecture activities for students by using analogy and generalizations in teaching Grade 11th geometry*, Journal of Science No.60, pp. 97-106, Hanoi National University of Education.

II. Paper in the National Science Conference

Vu Dinh Chinh (2014), *Training conjecture activities for high school students by using thinking method in teaching spatial geometry*, Proceedings of the National Science Conference "Mathematics education research in the direction of developing learners' ability, the period 2014 - 2020", Publishing House of Hanoi National University of Education, Hai Phong, April 2014.

PREFACE

1. Reason for choosing the topic

1.1. When we access Maths, it has been represented in a unified, completed manner, including pure proven steps. However, before the mathematicians find out how to prove a theorem or any definition or conception, they have to undergo the process of conjecturing it. Before starting to prove a theorem, they conjecture ideas of the proof. The mathematician's creative result is inferring proof, proving; but people find out the way to prove thank to reasonable inference and prediction. Conjecture is a necessary step of resolution.

1.2. Teaching and learning Maths not only merely equip students with the knowledge and skills but also through learning activities approach the development of intellectual activities, such as: analysis, synthesis, comparison, analog, generalization and specialization the teachers gave conjecture in the teaching and learning activities will ensure the development of a number of intellectual activities mentioned above. When the students conjecture the hypothesis or proven matters of the problem, the learners skillfully used intelligent steps to achieve the requirements of conjecture.

1.3. Today, at high schools, teachers introduce active teaching methods into teaching and learning process, such as: creative, resolution, discovered teaching… We have the same idea is resolution and discovered teaching. So the teachers impove capacity for conjecture to guide their students access to the above active teaching methods as well as the development of Maths competency. In parallel with training of conjecture capacity for students, teachers also train sound reasoning. These two activities are closely related to each other. If conjecture is not reasonable, it will lack of credibility and not be recognized. In contrary, if conjecture is reasonable but learners are not experienced, it leads to the imposed and one-way teaching.

1.4. In Mathematical programs at high schools, geometry is the important content to complete knowledge of Maths and develop students' thinking at high school. When studying this content, it is difficult for students at high schools to absorb by abstract knowledge, the ability of geometric inference is limited. The imagination about space of learner is blurry and not adaptable with the change from geometric to space geometric. Teachers' training in conjecture capacity and sound reasoning is the way to help students overcome those problems. In contrast, geometric content also contain appropriate elements to train students' conjecture capacity and sound reasoning in teaching process.

1.5. When teaching geometric problems, most of teachers often give the problem and ask students to prove by deductive method, this make students passive in geometric, understand the content unclearly and they are afraid of studying this subject. Teachers introduce conjecture activities into teaching helped them access the lessons actively, provide opportunities to create exercises

accordingly to each student's capacity. Students' conjecture can be right or wrong, so after judging new knowledge, they need to retrun to deductive inference to prove theirs are right or wrong. Thus, the practice of conjecture capacity is not separate from proof inference, sound reasoning.

2. Overview of Research Issues

G.Polya said that solving and judging the problem are two important activities of Mathematics. He gave the examples of analysing conjecture process through specialization and generalization role in mathematical activities. Polya said: Features of proving and finding the problem: Proving problem is to confirm the conclusion, comment without any ambiguity, either true or false. The confirmation consists of two parts: theory is accompanied by the word "if" and the conclusion is accompanied by the word "shall".

Conjecture is considered as the backbone of learning and researching Maths. Its importance was confirmed by Polya long time ago through stories about the history of Maths written in the book, "Mathematics and plausible inference." Besides formal inference, informal inference such as intuition, moment "suddenly flashed," ... always travel with the mathematicians on the journey to discover new knowledge. In short, much of the works of mathematicians do not look for ways to prove the existed clause was stated.

Some other educational scientists had many significant contributions of conjecture research, such as: Fischbein (1987), Arzarello (1998), Mason (2002), Furinghetti and Paola (2003) and Bergqvist (2005). Fischbein (1987) considered conjecture as the perceptual performance. Mason (2002) has demonstrated the importance of "conjecture environment". Conjecture environment encourages learners to find examples and counter-examples to support conjecture, it creates opportunities for learners to adjust and expand conjecture. Arzarello's work (1998), Furinghetti and Paola (2003) studied the role of "dynamic geometry environment" to support the students' conjecture. Arzarello study the use of the software "dynamic geometry" to create an environment to explore the geometry problems, conjecture, verify and confirm the correctness of conjecture. Furinghetti and Paola analyze the process that students find and verify their conjecture by "dynamic geometry" environment, the impact of the change from perceptual to abstract awareness. The author Bergqvist (2005) analyzed how to verify conjecture and how to make teachers believe it is related to the implementation process. The author John M. Gillis (2005) completed the study with the name of the project: "Study of the students' conjecture in static and dynamic geometry environment." In his research the author has designed "static and dynamic geometry environment" to help students to judging. Students create conjecture in "static geometry environment" is used to compare with conjecture in "dynamic geometry environment" through geometry software.

Some conjecture studies from Australia, Canada, Spain and Ukraine focus on answering the following questions: What types of conjecture and what

stages does conjecture include? What problem can you put in to develop conjecture?; How do we describe the characteristic capacity of each type of conjecture?

Canadas and his colleague group would like to mention the issues relating to the following questions: How can teachers teach students to create conjecture?; Why do not most teachers promote conjecture activities in class?; What obstacles in the process of teaching and learning conjecture?

Referring to type of conjecture, conjecture stages and how the problem can be developed by conjecture, Canadas and his colleague group synthesized a number of familiar conjecture in the study of Mathematical education; namely: induction conjecture from a limited number of individual cases, analog conjecture, generalization conjecture and perception conjecture.

The research group of Fou - Lai Lin (2006) showed that the conjecture activities can improve the proficiency of learning Maths. Kilpatrick's work and his colleagues (2001) have clarified Fou - Lai Lin’s idea by proposing five components:

The 1st: Speaking of the understanding of the concepts, operations as well as the relationships between concepts.

The 2nd: Refer to the skills in the implementation of the process flexibly, accurately, efficiently and reasonably;

The 3rd: Predict the role of stimulating the development of strategic; expression, presentation and resolution capacity of students. When facing the problem, if not imagine what it is, the first thing is restating the problem so that we can apply Maths to solve. Therefore students can give series of predictions about resolution and then gradually get in the core solution of the problem by the intellectual activities;

The 4th: Refer to the logical thinking ability, explain and justify. When students give conjecture, it is important to verify wheter the prediction is true or not. If wrong, just a counter-example is sufficient to refute conjecture;

The final component is describing trends that look for the meaning of Maths, think about it as a useful subject, have faith that all the efforts in learning Maths will receive worthy compensation for the learners themselves. When taking part in the tasks of conjecture, students have the opportunity to gradually develop their research capabilities. They will have to grope, observe and compare the results, apply the learned knowledge to solve the problem. Compared with any results "falling from the sky", proving a result that they conjectured will enhance the confidence of the learners themselves. Also, verify the correctness of conjecture help them experience the beauty and meaning of Maths. Since then the children will develop a positive attitude to learn Maths.

However, most of the above studies lack of clarity for what is discussed and conjecture is solved how exactly, and how to link to popular situations in

Maths education. Thus, it is clear that conjecture and resolution are two related activities, but the studies have not clearly shown. On the other hand, not all of the proplems lead to conjecture and different Mathematical problems lead to different types of conjecture.

In our country, the author Nguyen Van Loc (1992) has studied sound reasoning with the topic: "Forming skills of reasonable reasoning for first grade students at high schools in Vietnam through geometry teahching". The author's work emphasizes the important role of the logical analysis skills of the definition, concept, geometry clause and mathematical inference. The study aims to shape skills of basic sound reasoning and their relationship.

3. Purposes of the study

+ Clarify some basic expressions of conjecture capacity and sound reasoning to reveal knowledge in geometry teaching at high schools;

+ Recommend practical methods of conjecture capacity and sound reasoning for students in geometry teaching at high schools.

4. Subject of the study, object of the study and scope of the study

4.1. Subject of the study

The students' conjecture capacity and sound reasoning in geometry teaching at high school.

4.2. Object of the study

Through teahching process to reveal geometry knowledge at high schools

4.3. Scope of the study

Train conjecture capacity and sound reasoning for students through pedagogical measures in geometry teaching at high schools.

5. Scientific theories

Based on specific subject, students' cognitive characteristic and innovation requirements, if pedagogical measures are proposed to train conjecture capacity and sound reasoning in geometry teaching at high schools, that will strengthen creative, positive, active and capable inference of Maths in general and geometry in particular.

6. Research questions

- What scientific basis is based on to determine the basic expression of conjecture capacity and sound reasoning in geometry teaching at high schools?
- What is the scope of using conjecture and sound reasoning activities in geometry teaching at high schools?
- What is the scientific basis to determine the component activities of conjecture capacity and sound reasoning in geometry teaching at high schools?
- What are basic steps to help students with conjecture and sound reasoning?

7. Research methods

We mainly use the following research methods:

+ *Group of theoretical research methods*

+ *Group of practical research methods*

8. The contributions of the thesis

8.1. Theoretical contributions

- The thesis systematize some problems of conjecture and sound reasoning
- Build up practical measures of conjecture capacity and sound reasoning for students in geometry teaching at high schools through enhancing the positive for learners.

8.2. Practical contributions

- Measures taken to improve the geometry teaching and learning quality at high schools to enhance the positive for learners;
- Examples are included in the pedagogical experiments are teachers' references to train conjecture capacity and sound reasoning for learners at high schools.

9. Issues to defense

- The basic expression of conjecture capacity and sound reasoning in geometry teaching at high schools.
- The feasibility and effectiveness of the measures to train conjecture capacity and sound reasoning in geometry teaching at high schools.

CHAPTER 1: THEORETICAL FOUNDATION

1.1. Some terms relating to conjecture:

**Conjecture – Prediction – Hypothesis*

* *Conjecture capacity:* Conjecture capacity is activity capacity of the writer to find new hypothesis based on available knowledge and experience. "To get conjecture capacity of revealing problems, students need to be trained capacities related to pre-logical and dialectical thinking capacity. It was the capacity considers mathematical objects, the mathematical relationship in the relationship between the general and the specific, understand the causal relationship, there should be thinking actions, such as: Comparison and analysis , synthesis, specialization, generalization, and objects association capacity, known relationship with similar objects, similar relations. The degree of conjecture capacity closely related to the quality of creative thinking capacity, logical thinking capacity, critical thinking capacity, dialectical thinking capacity, resolution capacity, ...

* *The distinction of conjecture – prediction – hypothesis:*

- *Conjecture and hypothesis:*

+ *Similarity*: Both statements are based on observation and seem to be true but this is unproven and untested.

+ *Difference*: G. Polya and L.Bowden explained at the end of book "Mathematical methods in science," they said the difference of conjecture and hypothesis are based on trust level that its interpretation. Hypothesis can be verified and explanations based on an acceptable basis. Conjecture is proposed based on an "indefinite" basis and sometimes it has not been fully tested. Conjecture not the same with hypothesis, it is always used in Maths, hypothesis is used for all fields. If conjecture is true then it is the theorem. However, the boundary between these two terms are still unclear, sometimes they are used as synonyms. For example, in Maths, conjecture is called "Riemann hypothesis" but exactly called "Riemann conjecture".

- *Conjecture and prediction:*

Conjecture has a logical structure, based on certain grounds to confirm or deny any signal of the object. Prediction has no logical structure, prediction is just conjecture, unfounded so the reliability of the conjecture lower than prediction.

* *The relationship between conjecture and resolution*

Canadas with his colleagues pointed out: resolution and conjecture are two extremely realted activities. However, not any kind of problem is also a fertile ground for conjecture arises. According to Yevdokimov’s classification, there are closed and opened problem. The closed problem is similar to proven problem in Polya’s classification. This kind of problem is similar to the kind of research problem of Polya’s classification. However, we can give diverse situations for the prediction, thereby develop students’ mathematical inference. On the other hand, according to De Villier’s opinion, teachers should use and alternate between research problems and proven problems reasonably.

1.2. Sound reasoning – Inference

**Sound reasoning*

Reasoning is sort reasoning in a systematic way to present, to prove a conclusionof problem. Basis is "based on", "as a foudation" to argue or act. The basis of the reasoning that the known axiom, theorem, characteristic and consequence, definition and available hypotheis of mathematical problem.

** The relationship between conjecture and sound reasoning:* conjecture and sound reasoning are not independent, but we have close relationships with each other. Conjecture finds out resolution, the premise for each reasoning step. Conversely, sound reasoning exams, revises whether the premise is right; rates inference rule used in mathematical problem solution. These two activities complement to each other in the process of finding solution and checking answer.

** Inference – deduction – reasonable inference and "sound-reasonable inference"*

** Inference rule*

** Prove*

** Inference in geometry*

1.3. The basic expression of conjecture capacity

+ Capacity considers mathematical subjects, mathematical relationships in the relationship between the general and the specific.

Conjecture way through the relationship between the general and the specific can be done through the following steps: interactive survey in specific cases; analysis activity, comparison and synthesis find out the relationship between the component elements; generalization activities give the general (conjecture activity); verify and give characteristic, the general rule (if true) in new situations.

+ Capacity of using intellectual activities seek to conjecture new mathematical problem

** Analysis and synthesis*

We understand that conjecture was formed by intellectual activity, analysis and synthesis is the addition of the missing premises for the conclusion of mathematical problem. The addition premise of each student is different because of the difference of each student's mathematical capacity.

** Comparison*

Comparison has two purposes: reveal the general characteristics and the different specific characteristics of an object, event. The first aim often leads to similarity and parallel with generalization. Thus, in order to conjecture an object is similar to known object, the learners use comparison skill, comparison to find out the similarity between two objects, from which come to conclusion that the first object has any characteristics, then the second object has those characteristecs.

** Generalization*

Students' conjecture capacity expresses through the capacity to find the relationship of specific case to predict the generalized case.

** Specialization*

The role of specialization become important in case we are having prediction on considered object and would like to prove it is right, but we don't know how to prove. In that case, we should use specialization. Applying prediction in a few special cases, if this our prediction is right, then ours is more reliable.

+ Capacity of associating objects, known relation with similar objects, similar relation

Conjecture based on association is very important in resolution. Sometimes in expansion process, the set of objects that have similarity, from which we infer the characteristics from this group to another group.

+ Object association capacity to detect and solve new situations

Conjecturement capacity shown in capacity of association between objects to detect and solve new situation.

+ The capacity to detect rules or mathematical characteristics by using inductive inference.

We can conjecture a clause through observing some specific cases which have a relationship. In some cases, that conjecture is proved by inductive Maths as a general rule found. Thanks to the use of thinking actions: analysis, synthesis, specialization, generalization to give conjecturement about object, relations or mathematical rules.

+ Capacity of using extrapolation to choose the best explanation for the problem

+ Capacity of using mathematical performance to explore the rules or mathematical characteristics

One conjecture can be made from the perception idea of the problem or conversed perception of the idea. The basis of this conjecture is the content performance of problem specifically or intuitive thinking. Once this performance exists, it is strong enough to accurately perform the relationship of mathematical elements of the problem, therefore support both conjecture and proof.

1.4. The basic expressions of sound reasoning of students.

+ Ability to analyze the logical structure of the problem. From which they look assumptions and conclusions of the problem in all other respects

The analysis of the premises and the conclusion of problem to reveal signs of nature, the relationship between the signs are considered, the contrasting images of the signs, finding different expressions of the concepts under study.

From the analysis of the premise (right clause) and the conclusion of the problem to help students to practice the premise look and concluded in

accordance with different perspectives based on theorems, axioms or concepts that they have learned . Since then they built a lot of different theories from a base case.

+ Ability to see solution of problem through diagrams "downward analysis"

To find right ways, students can analyze the scheme goes down as follows: $B \rightarrow A_1 \rightarrow A_2 \rightarrow ... \rightarrow A_{n-1} \rightarrow A_n$. This diagram is not proven that it helps students to find problem-solving way.

+ Ability to identify the foundation of each step in problem solving of students

It is likely that students use the base of each step in own the mathmatic problem resolving presentation to confirm the rightness of the problem.

+ Ability to test, evaluate solutions of the problem based on inference rules

It is the ability to understand and apply inference rules on solving problems, the inference rules: The common types of syllogism (assertion, negation, choice, bridging); conclusions rule from common clause; inductive inference rule.

+ Ability to find counter-examples to refute propositions

One clause stated unknowningly it is right / wrong. If want to reject propositions, we only need to find a counter-example to prove this statement wrong.

1.5. Scope for using well- grounded conjecture and reasoning in geometry teaching at high school

**** Teaching concepts***

To students may itself state, students should examine individual cases, through intellectual activities such as: Analysis - synthesis, comparison, generalization, specialization, abstraction, .. to draw signs essence of the concept and the notion that speech.

**** Teaching theorem***

Theorem Teaching has conjecture step is shown in the following steps:
Suggests Theorem learning; theorem predictions and statements; theorems proving; theorem using

* ***Teaching exercises***

We propose a way to find the solution for student to find a geometry problem solution as follows: Find out content of problem; predict the solutions of the problem (Problem solving by conjecture and by road using well- sound reasoning); presented the solution of the problem; research solutions.

1.6. Design conjectures

+ The principles of conjecture designing

Observing principle: Students have the opportunity to observe the system of individual cases to be able to discover any rules thanks conjecture

Construction principle: The tasks of judging create conditions for new knowledge building based on old knowledge.

Changing principle: Conjecture mission should be provided to students the opportunity to change the algorithms, rules, statements, formulas ... into a new

one. Such as change the premise or the conclusion of a speech, change one or several factors in certain formulations.

Considerations Principle: The following questions are some suggestions to ensure the principle of discretion in the task designing for conjecture: "Has my conjecture cleared yet?", "You explain why you believes it is right conjectured in accordance with the given conditions? "," is it possible to generalize the conjecture of you? "," do you believe your conjecture remains true when a certain condition is changed? ". To answer this question, the student must have a scientific basis to explain, that mean you prove your conjecture, you can find counter-examples during explanation to show my conjecture it still not fair, since they can change the conjecture accordingly.

+ Design of conjecture

Fou-Lai Lin had raised suggestions for the design of the judging duties. According to the author, we can start teaching using conjecture from one of the following two starting points: From a true statement; from a conjecture of the learner.

CHAPTER 2
RESEARCH SURVEY

2.1. The purpose of the survey

The design study aimed at the following:

Research practical basis of research problems that are not only based on the results of the previous study; aware of the difficulties they often encounter when learners are in conjecture and sound reasonings; building measures for student to train conjecture capacity and sound reasoning (measures will be proposed in Chapter III) based on practical of research issues towards the measures will overcome the difficulties they often encounter.

2.2. Subjects participating in the survey

+ Select classes in grade 11: specialized in Physics, specialized in Literature, specialized in Russian and 10th grade specialized in Math of Nguyen Hue Specialized High School - Ha Dong - Ha Noi. Each class has 30 students, the capacity of their mathematics in every grade is relatively uniform.

+ Math Teachers of specialized high schools Nguyen Hue - Ha Dong-Hanoi.

2.3. How to organize the survey

+ Before conducting the survey we have acquainted with class.

+ We designed the content for students to conjecture in class time and class is divided into smaller groups so that they can discuss and debate with each other. According to our opinion, the size of each group from three to four is appropriate because this number is enough to create atmosphere of discussion .. In addition, we also consult with Math teacher and homeroom teacher on the

distribution group to suit the their capacity, character and relationships so that there is harmony between the groups.

2.4. Survey duration

The survey was conducted from April 2nd, 2014 to April 29th, 2014

2.5. Survey tools

Survey tools that we design included: learning poll; Questionnaires for teachers; Interview questions for students.

2.6. Data collection and evaluation criteria

+ The analysis criteria for each step of grounded conjecture through inductive

+ The analysis criteria for each step of grounded conjecture by similar reasoning

+ The analysis criteria for each step of grounded conjecture by generalizing

+ The analysis criteria for each step building of a problem hypothesis by considering the assumptions and conclusions under different aspects

2.7. Survey result

2.7.1. Results of the questionnaire for teachers

+ When teaching the knowledge of Mathematics, 25% of teachers organizes activities that use conjecture to the extent sometimes, they are not aware lesson plans for training Conjecture activities. However, 65% of teachers has expressed interest in the ability of the student's conjecture and found them to be helpful in the teaching and learning activities.

+ When teaching concepts and theorems, majority of teachers teach students to focus directly manipulate the concept, theorem on solving exercises in textbooks, advanced exercises.

+ When teaching the exercise, 30% of teachers are not encouraging students to find many different solutions of a problem.

+74% of teachers have used the forms change actions to regulate the problem of familiar strangers, generalizing the problem.

+ When teaching mathematics, 61% of teachers are only interested in teaching Maths knowledge that students are learning, not pay attention to the development stage of knowledge of Mathematics which is taught.

+ There are 68% of math teachers in high schools only pay attention to serve the examinations, has not really focused on training conjecture capacity and sound reasonings

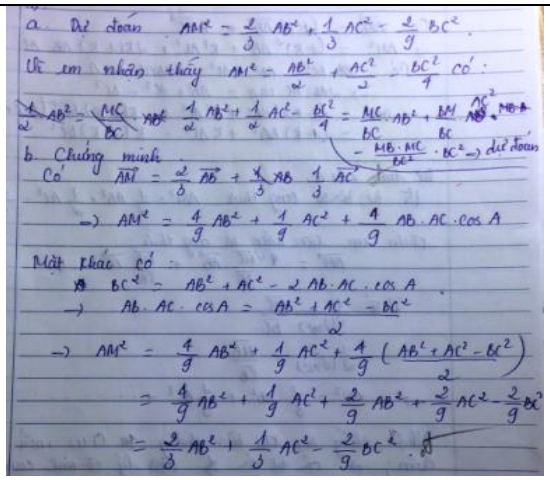
+ In mathematics teaching, 65% of teachers were interested in training for students capacity to solve the space problem into flat problem.

+ When teaching about content of space geometry, 72% of the teachers were paying much attention to the link between flat and space geometry.

From the above, we can notice that: The training for students for conjecture capacities and sound reasonings in the High School, teacher was initially interested but lacking of systematic and sync, no proper awareness of the significance and importance of it.

2.7.2. The discussion results and assignments of students through experimental sessions

Student performed handout 1

assignment of Group 3	Conjectures process analysis
	The prediction of the third group is quite favorable and easy. They said they anticipate the fast recipe because they are based on the formula: $AM^2 = \frac{MC}{BC}AB^2 + AC^2 - \frac{MC.BM}{BC^2}.BC^2$. They also proven formula coincides with the results they had expected. This makes them very excited about the work you did.

Surveying is perform with handout 2

Posts made by some group of similar conjectures and verified by judging by proven knowledge of Mathematics:

We noticed a lot of students have found similarities between the problems with their own reasonings under the following trends:

Trend 1:

Group 1,3,4,5,7 answers they think the concept of the triangle in a plane is similar to the concept of non-tetrahedral

Trend 2: Group 8, 9, 10: Unexplainable but they also said that the concept of the triangle in a plane is similar to the concept of space tetrahedron.

Trend 3: Group 2 and 6 explain the similarity of the two problems as follows:

+ *The concept of the triangle in a plane is similar to the concept of space tetrahedron*. In the course of the survey we received analyze the similarity between the triangular and tetrahedral from them as follows:

• Fold one equilateral triangles can form a tetrahedral

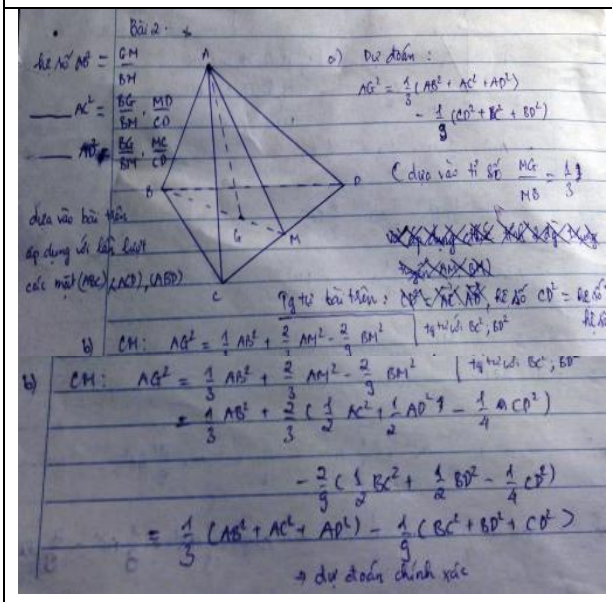
• They put their questions to the group:

"Can we fold a given triangle into a tetrahedron or not?"; "With the right-angled isosceles triangle how?". "Can triangular be folded into the tetrahedral or not?"

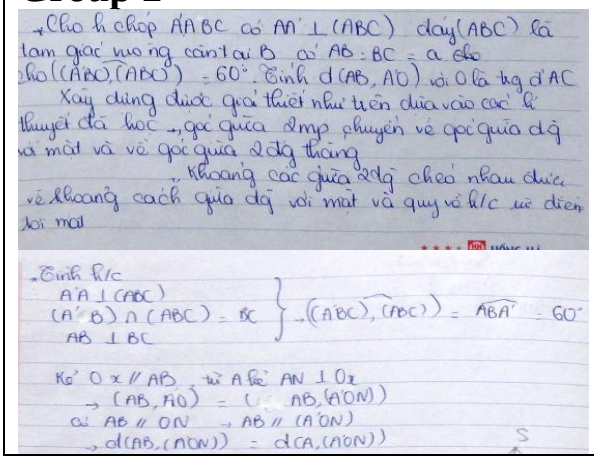
• From which, they note that such concepts of triangle in flat geometry is similar to the concept of space tetrahedron.

+ 10 groups find out that: M is the midpoint of BC, $\overrightarrow{MB}+\overrightarrow{MC}=\vec{0}$ and H is the focus of the triangle ABC, $\overrightarrow{HA}+\overrightarrow{HB}+\overrightarrow{HC}=\vec{0}$. *Thus the concept of the midpoint of the line is similar to the concept at the heart of the triangle ABC.*

+ All of these groups do predict and verify the prediction for formula to caculate AH:

Result of group 2	Conjectures process analysis
	The group also predicted similar to the results of other groups and explain how the following predictions: Coefficient of AB^2 is: $\frac{HM}{BM}$ Coefficient of AC^2 is: $\frac{BH}{BM}.\frac{MD}{CD}$ Coefficient of Similar to prove the group on their part have the same results with anticipation. AD^2 is: $\frac{BH}{BM}.\frac{MC}{CD}$ Coefficient of BC^2 is product of the coefficients $(-1).AB^2.AC^2$ Similar to the coefficient of CD^2, BD^2 similar reasoning BC^2. Similar to prove the group on their part have the same results with anticipation.

Survey performed with handout 7

Group 2	
	Group 2 also developed similar hypotheses other groups. They look at the distance from point A to (A'MN) is the distance between two straight lines AB and A'O cross.

2.8. Difficulties that students often encounter when conducting conjectures and theories setting

The difficulties when they conduct conjecture:

After the students proceed and complete the contents of the questionnaire, we ask the question for them around themes of conjecture. The question we really want answered them: "The children have difficulties when conducting conjecture operations?". Many of them answered as follows:

+ When you start to conjecture a certain problem, they do not know on what basis to predict due to difficulties in raising capital their existing knowledge with the knowledge to conjecture.

+ They're used to that teachers put available hypothesis problem and requested proof. When working with a formula to predict or construct theories to a problem, then they feel embarrassed for not knowing how to do and where to start. However, after the first Conjecture activities, activities for the next conjecture has fascinated and their thinking with flexibility and independence.

+ Many of them answered that they are very embarrassing to explain why they have such predictions due to language limitations, including the language of mathematics, due to difficulties in the wording of the problem

+ Some children do not like to participate in predicting because when they participate in activities that will showcase conjecture several disadvantages such as the passive, difficulties in language expression, confusion in reasonings, poor ability of defending, lack of the skills to interact with partners, ...

• *The difficulties when students conduct hypothesis construction activities*

+ The children are confused between how solve the problem and how to build. However the formulation of hypothesis help them identify the core issue of the problem of expansion.

+ The construction of the hypothesis of the problem requires mastery of geometry, requires a lot of time to think.

+ Some of them just to find the simplest theory because they do not have flexible use of theorems, geometry definitions. Finding premises often require them to have knowledge about the geometry system and apply the sharpness of that knowledge.

+ This is a new way for them so they are still embarrassed while performing. Teachers need to provide more basic problems to help them work out the construction of hypotheses from this problem.

But for the children haveing good mathematical ability, they are interested in the basic problems hiding the important points and the crux of the problem, for children to explore and discover themselves.

CHAPTER 3
MEATHODS FOR FOSTERING STUDENTS CONJECTUREMENT CAPACITY AND GOUNDED REASONING IN TEACHING GEOMETRY AT HIGH SCHOOL

3.1. Meathod 1: Create situations for students to conjecture in teaching geometry using inductive reasoning and similar reasoning

+ Objectives

The meathod 1 with the aim to fostering student’s conjecture capacity through inductive inference and similar reasoning. The inclusion of inductive inference and similar reasoning in mathematics teaching was no stranger to students. However, thanks to inference that the students are trained to conjecture the work they do not often get to experience before. The conjecture of students in the class helped the children with a new view of mathematics learning, students see that the tight relationship between the conjecture to

demonstrate the problem, towards the construction of a problem solution through the use of new similar to the steps of a problem to prove familiar.The children not only learn mathematics for the goal of solving problem, but also worked as a mathematics researcher.

+ Foundation and the role of the meathod

+ Guidline of the meathod

- **Guidline students to conjecture by using inductive reasoning**

Step 1: *Teacher organizes for students to observe and conjecture in individual cases*

Step 2: *Students test the results that they just conjecture in individual cases*

Step 3: *Teacher guides student to arrange separate cases systematically*

Step 4: *Students search for and detect a general rule from individual cases*

Step 5: *Learners generalize conjecture*

Step 6: *Teacher guides students demonstrated in the general case*

- **Guidline students to conjecture by using similar reasoning**

Step 1: Teacher let students to compare two objects;

Step 2: Students seek similarities between two objects;

Step 3: The teacher suggests learners looking for a few other properties in the first object;

Step 4: Teacher let student to conjecture simmilar properties of the remaining objects;

Step 5: Students examine the true /false of conjecture based on mathematical reasoning.

Step 6: The students confirme their conjecture is right or wrong

Example: Described the hypothetical construction of spatial geometric problems follow a similar way and explore the solution of the problem based on the new way association based on similar problems in geometry plane.

The problem in plane geometry: "Circle of a triangle *ABC* with center *O*, straight line perpendicular to *AO* cutting *AB*, AC respectively at *M* and *N*. Prove four points *M*, *N*, *B*, *C* belong to the circle.

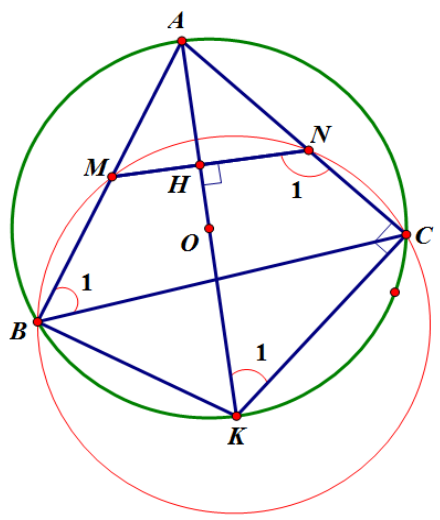
prove: Considering quadrilateral *HKCN* we have

$H = 90^0$ và $C = 90^0$ so $H + C = 180^0$.

$\Leftrightarrow N_1 + K_1 = 180^0 \Leftrightarrow N_1 + B_1 = 180^0$

($B_1 = K_1$ due to together blocking chord AC).

So, the points M, N, B and C lie on the circle.



From the students are to observe the object in pairs Circle - sphere; line - plane; Triangle - tetrahedron to detect some similar characteristics between them:

- The concept of the circle in a plane similar to the concept of spheres in space
- The concept of the triangle in a plane similar to the concept of tetrahedron in space.
- The concept of a straight line in a plane similar to the concept of a plane in space.

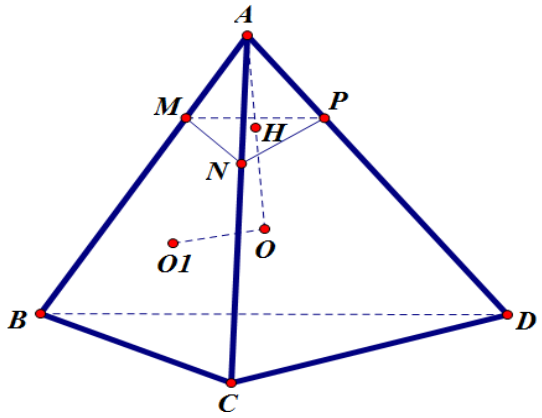
Specifically, similar objects together as follows:

Circle center *O*	Sphere center O
Triangle ABC	Tetrahedron ABCD
AO straight-line perpendicular to the cutting edges AB, AC respectively at M, N	AO perpendicular plane cutting edges AB, *AC*, *AD*, respectively at *M*, *N*, *P*.
M, N, B, C under a circle	**Prediction:** *M*, *N*, *P*, *C*, *C*, *D* together in one sphere

Therefore, we can predict the assumption of the new problem as follow:

Prediction of this problem in space geometry:

Sphere with center *O* circumscribed tetrahedron *ABCD*. A plane perpendicular to the line A cut edges *AB*, *AC*, *AD*, respectively at *M*, *N*, *P*. Proving that ***six point B, C, D, M, N, P are together in one sphere.***



To look for the answer for the above problem, the teacher helps students think of a familiar problem (theorem) learned in plane geometry and base on the characteristic: “On each plane in space, the known results in plane geometry are all correct”.

Theorem: “If the circle center *O* circumscribe triangle *ABC*, the perpendicular line to *AO* cut *AB*, *AC* respectively at *M*, *N*, four points *M*, *N*, *B*, *C* are together in one circle” (the theorem is used for form the hypothesis) is used to be the foundation to prove the problem in space geometry. Specifically:

Concerning *ABC* of tetrahedron *ABCD*, we have $M \in AB, N \in AC$. To apply answer of the problem in plane geometry, students need to supply the lack assumption which is: O_1 is the center of circumscribed circle of triangle ABC and $AO_1 \perp MN$.

Because *O* is the center of circumscribed sphere of tetrahedron *ABCD* so *OA*=*OB*=*OC*. Therefore, if O_1 is the perpendicular projection of O on plan (*ABC*), $O_1A = O_1B = O_1C$ and $AO_1 \perp MN$ (theorem of three perpendicular lines). Hence, if we

supply the hypothesis "O_1 is the perpendicular projection of O on plan (*ABC*)", O_1 is the center of circumscribed circle of triangle ABC and $AO_1 \perp MN$.

Applying results of the problem in plane geometry, we have: *M*, *N*, *B*, *C* are together on the circle (C_1). Similar to the above way, students can also prove that *N*, *P*, *C*, *D* are together on the circle (C_2).

Two circles (C_1) and (C_2) belong to two intersect planes which intersect at two points N, C. Because two circles (C_1) and (C_2) are in the sphere, six points *B,C*, *D*, *M*, *N*, *P* are together in one sphere.

+ Some notes when using the method

3.2. Method 2: Creating situations for students to conjecture in teaching geometry by using generalization

+ Objectives

Method 2 helps students in training conjecturement ability based on generalization from some particular things. This method helps student in forging the ability to generalize from one to two particular situations.

+ Foundation and the role of the meathod

+ Guideline of the method

Step 1: (*Observation*): Teacher organize for students to observe some particular situation (particular thing).

Step 2: (*Analysis*): Students discuss to analyze or compare the particular situations in order to find out the relations and characteristics of particular things in those relations.

Step 3: (*Conjecturement of generalized situation*): Students conjecture generalized situation (general thing)

Step 4 (*Verifying*): teacher guide students in verifying the prediction by mathematical inference.

Step 5: Learners confirm whether the conjecturement is true or false.

3.3. Method 3: Creating situations for students to conjecture in teaching geometry by using extrapolation

+ Objectives

By using method 3, students can practice conjecturement ability based on extrapolated inference. Using extrapolation in teaching mathematics helps students practice conjecturement. The conjecturement in learning helps students have a new view in learning mathematics.

+ Foundation and the role of the meathod

+ Guideline of the method

Step 1: A event (phenomenon, result,..) **S** is observed;

Step 2: Appearance of hypothesis **G** to explain for **S**;

Step 3: No hypothesis that explain for **S** better than **G**;

Step 4: Then **G** is the best explanation for **S**.

Example: Problem **S**: "There is a square with edge a. They cut at four angle, then fold it for a rectangular without cover. Predicting the length of the edge of

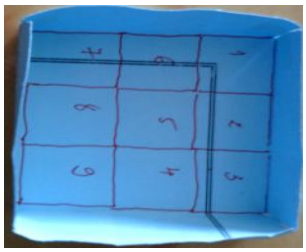
the squares which have been cut so that the volume of the rectangle is maximum? Using mathematical inference to verify your prediction".

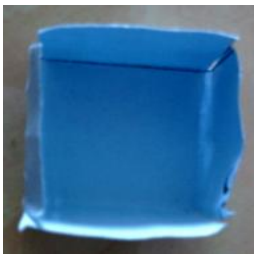
Step 1: Perform problem S in visual performance so that it can be observed in the best way.

Dividing class into several groups, the teacher gives each group the same square piece of paper. Giving students some tasks as follows:

+ Cut the four angle of that square some square with the same size. Predicting the length of the edge of the squares which have been cut so that the volume of the rectangle is maximum.

When practicing, some groups have folded the square into some rectangles as follows:





Step 2: There are several hypothesis including hypothesis **G** which explain for **S**

After folding, from some results, the groups have listed as in the following table:

Ways of folding	x is the edge of the squares have been cut at the four angles $\left(0 < x < \frac{a}{2}\right)$	Volume of the cube with edge length *x*	Volume of the rectangle (Total volume of of the cubes with edge length *x*)
Dividing the square into 12 equal parts	$x = \frac{a}{12}$	$\left(\frac{a}{12}\right)^3$	$64.\left(\frac{a}{10}\right)^3 \approx 0,0579.a^3$
Dividing the square into 10 equal parts	$x = \frac{a}{10}$	$\left(\frac{a}{10}\right)^3$	$64.\left(\frac{a}{10}\right)^3 \approx 0,064.a^3$
Dividing the square into 8 equal parts	$x = \frac{a}{8}$	$\left(\frac{a}{8}\right)^3$	$36.\left(\frac{a}{8}\right)^3 \approx 0,0703.a^3$
Hypothesis **G**: Dividing the square into 6 equal parts	$x = \frac{a}{6}$	$\left(\frac{a}{6}\right)^3$	$16.\left(\frac{a}{6}\right)^3 \approx 0,0741.a^3$
Dividing the square into 5 equal parts	$x = \frac{a}{5}$	$\left(\frac{a}{5}\right)^3$	$9.\left(\frac{a}{5}\right)^3 \approx 0,072.a^3$
Dividing the square into 4 equal parts	$x = \frac{a}{4}$	$\left(\frac{a}{4}\right)^3$	$4.\left(\frac{a}{4}\right)^3 \approx 0,0625.a^3$
Dividing the square into 3 equal parts	$x = \frac{a}{3}$	$\left(\frac{a}{3}\right)^3$	$\left(\frac{a}{3}\right)^3 \approx 0,03703.a^3$

Step 3: Students discuss to predict that there is not any hypothesis that explains for **S** better than **G**

After listing the hypothesis to explain for the problem **S**, students think that hypothesis **G** is the best explanation for problem **S**.

Step 4: Predicting that G is the best explanation for S. After testing, some groups have predicted that with $x = \frac{a}{6}$ then the rectangle will have maximum volume $V_{\max} = \frac{16a^3}{216} = \frac{2a^3}{27}$; and they believe that their prediction is the best solution for the problem S.

Step 5: Using sound reasoning to verify that hypothesis G is the best solution.

3.4. Method 4: Creating situations for students to conjecture in teaching geometry by using mathematical performance

+ *Objectives*

By using method 4, students can practice conjecturement ability based on mathematical performance, which helps students recognize the close relation between conjecturement and visual performance of the problem. In the process of visual performing, students can use some geometric software to handle and verify their prediction so that they can strengthen the belief in the accuracy of their prediction.

+ *Foundation and the role of the meathod*

+ *Guideline of the method*

Step 1: The teacher guides students to transfer the problem in to visual performance (picture, model,…);

Step 2: The teacher organize for students to observe characteristics of the performance related to the problem need to be solve;

Step 3: Students use mathematical software to take action, calculate, measure, draw some sub lines…. to support for predicting;

Step 4: Students predict the conclusion of the problem based on the results;

Step 5: The teacher guides students to use mathematical inference to confirm that their prediction is right.

3.5. Method 5: Having students practice in choosing correct premise for solving problems

+ *Objectives*

Method 5 helps students in practicing to choose the correct premise to solve the given problem. In order to choose the right premise, learners need to be able to recognize conclusions from the given premise or to analyze under downward schema to find out way for proving.

+ *Foundation and the role of the meathod*

+ *Guideline of the method*

Step 1: The teacher analyze proving process of the problem in order to look for the correct premises used in solving that problem.

Step 2: The teacher has students summarize the right premises used in solving problem.
Step 3: The teacher has student present answer for the problem (based on the chosen correct premises)
Step 4: The teacher gives students some exercises for them to consolidate and practice.

3.6. Method 6: Training students to have the habit of checking, evaluating again steps of reasonings. Comparing different solutions for the same problem

+ *Objectives*

The objective of this method is helping students form the habit of checking, evaluating again their steps of reasonings after presenting the answer. Moreover, students should compare their solutions with others in order to find the best one. By using this method, learners understand the inference rule in each solution.

+ *Foundation and the role of the meathod*

+ *Guideline of the method*

Step 1: The teacher guides student groups to solve a problem in different ways.
Step 2: The teacher guides students to find out the ground of any single step and inference rules using in various ways of answer.
Step 3: The teacher have students evaluate the ground using in the steps of every answer and compare answers which give the same result.
Step 4: The teacher gives other situations for students to practice the method.

3.7. Method 7: Forging sound reasonings for students by concerning hypothesis and conclusion in different views

+ *Objectives*

The objective of method 7 is forging sound reasonings for students through analyzing hypothesis and conclusion, then concerning that hypothesis and conclusion in different views based on the ground of the reasonings. Besides, the fact that students can build up hypothesis for a problem from an original one also helps them deeply understand about geometric characteristics, relations between geometric subjects, flexibility in manipulating theorems, premises or definitions in geometry.

+ *Foundation and the role of the meathod*

+ *Guideline of the method*

Step 1: The teacher guide students to divide a part of a geometric shape out of the subject without changing any hypothesis or conclusion of the problem.
Step 2: Students, by themselves, build up hypothesis for the new problem by changing the hypothesis and conclusion of the problem by an equivalent thing.
Step 3: Students state the problem after completing.
Step 4: Students solve the given problem to find out the reasonability of the problem. In this step, students can also predict the solution by some geometric factors or linking to the familiar problem….
Step 5: The teacher confirm or reject the hypothesis built by the students.

CHAPTER 4
PEDAGOGIC PRACTICE

4.1. Objectives, requirements, content of the practice

4.1.1. Objectives

Initially surveying, evaluating conjecturement ability and sound reasonings in teaching geometry at some high school in Ha Noi city; concerning possibility and effectiveness of the suggested pedagogic methods in order to verify scientific hypothesis of the dissertation.

4.1.2. Requirements

Pedagogic practice must ensure objectivity and appreciation to students and real situation in teaching geometry at high school currently.

4.1.3. Practice content

In order to have common suitable objectives for most of students, we base on some foundations: pedagogic aims; amount and level of knowledge, skill that students have on the basis of surveying by questionnaire, observation; based knowledge standards specified in professional guidance document.

4.2. Time, process and method of pedagogic practice

Based on specific requirements of the dissertation, we proceed in two rounds:

Round 1: Academic year 2013 – 2014, first practice in four classes: The practiced classes: $11A_2$, $11A_4$ and reference classes $11A_1$, $11A_3$ at Ha Dong High School, Hanoi.

Round 2: Academic year 2014 – 2015, second practice in two practiced classes: $11A_2$, $11A_4$ and two reference classes $11A_1$, $11A_3$ at Tay Ho High School, Hanoi.

Round 1 aims at verifying lesson design according to general approach. Verifying teaching organization. Verifying teaching using methods of practicing conjecturement ability and sound reasonings for students at high schools. Verifying communicated environment of students at high school; Then, proceeding amendments for round 2 in order to verify synchronization application of methods of practicing conjecturement ability and sound reasonings in teaching geometry at high schools for students in different environments.

4.2.3. Methods of pedagogic practice

4.2.3.1. Elevation content

Effectiveness of using methods of practicing conjecturement ability and sound reasonings for students to the response for pedagogic aims by proceeding lessons is evaluated based on:

4.2.3.2. Methods for pedagogic practice evaluation

+ ***Writing test:*** Aims at evaluation level of conjecturement of students through every lesson. Testing conjecturement ability and sound reasonings of every individual in the practiced class and reference class by the writing test.

+ ***Handouts for students:*** in order to evaluation level of awareness, capture and demonstrate the use of fostering measures for conjecturement ability and sound reasonings for students, we use handouts.

+ ***Interview teachers and students after practice:*** In order to have information in effects of using fostering measures for conjecturement ability and sound reasonings for students in teaching geometry, we use the method of interview to clarify information.

+ ***Mathematical statistics methods***

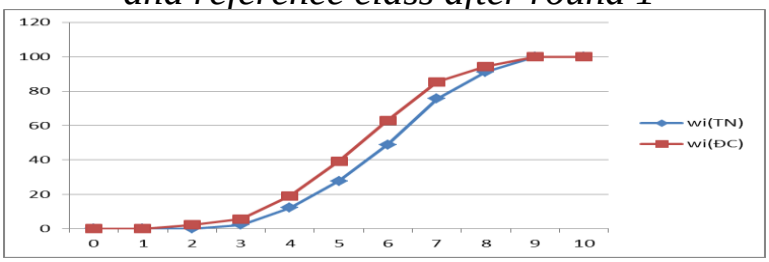
4.3. Process of pedagogic practice

4.3.1. Pedagogic practice round 1 *(Academic year 2013 - 2014)*

Mark distribution of practiced and references class after practicing round 1 in grade 11

x_i (mark)	Total number of students	0	1	2	3	4	5	6	7	8	9	10
f_i (*Practiced*)	90	0	0	0	2	9	14	19	24	14	8	0
f_i (Reference)	89	0	0	2	3	12	18	21	20	8	5	0

Diagram shows cumulative back-converge frequency line of practiced and reference class after round 1



Using t-student test to concern effectiveness of pedagogic practice, we have $t = \sqrt{\frac{\bar{x}}{S_{TN}}} = \sqrt{\frac{6.42}{1.51}} \approx 2.06$, looking up in the t-student distribution table, t = 2.06 > 1.67 = t_α. The practice has markedly results.

Proceeding verifying variance between practiced and reference classes with the hypothesis E_0: *The difference between variances in practiced and reference class is meaningless.*

Verified quantity: $F = \frac{S_{DC}^2}{S_{TN}^2} = \frac{2.52}{2.29} \approx 1.1$

Hence, $F < F_\alpha$: E_0 is accepted, which means that the difference between variances in practiced and reference class is meaningless.

In order to compare the practice results, we verify hypothesis H_0: *"The difference in average mark between two samples is meaningless in case of the same variance".*

Calculating verified value: $t = \frac{\overline{x_{TN}} - \overline{x_{DC}}}{\sqrt{\frac{(N_{TN}-1)s_{TN}^2 + (N_{DC}-1)s_{DC}^2}{N_{TN}+N_{DC}-2}} \cdot \sqrt{\frac{N_{TN}+N_{DC}}{N_{TN}.N_{DC}}}} \approx 2.35$

We have $t \approx 2.35 > t_\alpha = 1.96$, which confirms that the hypothesis H_0 is rejected. Then, we can prove that the difference in average mark between two samples is

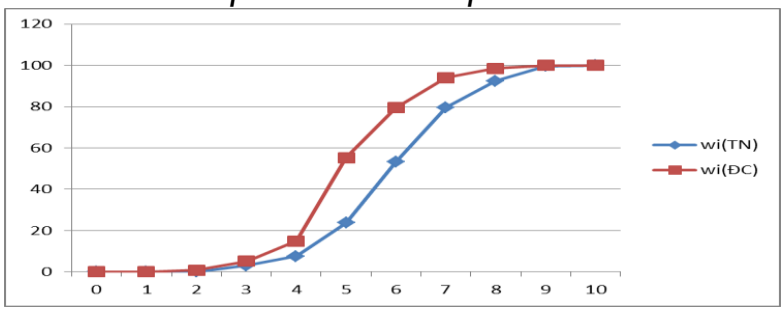
meaningful. The verified results show that the results of practiced class are better than that of reference class.

4.3.2. Pedagogic practice round 2 *(Academic year 2014 - 2015)*

Mark distribution of practiced and references class after practicing round 2

x_i	Total number of students	0	1	2	3	4	5	6	7	8	9	10
f_i (*Practiced*)	265	0	0	0	8	12	43	78	70	34	19	1
f_i (Reference)	268	0	0	2	12	26	108	65	39	12	4	0

Diagram shows cumulative back-converge frequency line of practiced and reference class after round 2



Using t-student test to concern effectiveness of pedagogic practice, we have $t = \sqrt{\frac{x_{TN}}{S_{TN}}} = \sqrt{\frac{6.41}{1.39}} \approx 2.15$. Therefore, t = 2.15 > 1.96 = t_α. The practice has markedly results.

Proceeding verifying variance between practiced and reference classes with the hypothesis E_0: *The difference between variances in practiced and reference class is meaningless.*

Verified quantity: $F = \frac{S_{TN}^2}{S_{DC}^2} = \frac{1.94}{1.55} \approx 1.25$

Hence, $F < F_\alpha$: E_0 is accepted, which means that the difference between variances in practiced and reference class is meaningless.

In order to confirm the quality of practiced and reference class, we verify hypothesis H_0: *"The difference in average mark between two samples is meaningless in case of the same variance".*

Calculating verified value:

$$t = \frac{\overline{x_{TN}} - \overline{x_{DC}}}{\sqrt{\frac{(N_{TN}-1)s_{TN}^2 + (N_{DC}-1)s_{DC}^2}{N_{TN}+N_{DC}-2}} \cdot \sqrt{\frac{N_{TN}+N_{DC}}{N_{TN}.N_{DC}}}} \approx 7.75 > 1.96.$$ The hypothesis is rejected, which proves that studying results of practiced group is better than of reference group.

CONCLUSION

The dissertation clarifies basic performances of conjecture capacity in teaching geometry at high school including: conjecturement ability in recognizing problems and in problem solving by using relation between the general and particular thing; intellectual activities; similar inference; inductive inference; Mathematical extrapolation and demonstration.

The dissertation defines basic performance of sound reasonings in teaching geometry at high school including: ability in analyzing logical structure of the problem; ability in looking for solutions by using "downward analysis" diagram; ability in defining ground for every step of reasoning; ability in checking and evaluating answers; ability in looking for contrast example to reject the clause.

The dissertation clarifies useful scope of conjecturement activities and sound reasoning in teaching geometry at high school, which is stated through particular teaching situations: teaching concepts, teaching theorems and exercises in the mathematics program at hight school: teaching vector geometry at grade 10, teaching space geometry at grade 11 and grade 12.

The dissertation defines scientific basis for determining component activities of structural activity of conjecturement and sound reasoning in teaching geometry at high school is rules of dialectic intellectual: "reasonable inferences"; pre-logical intellectual; ability in linking and moving linking; geometric premises; logical knowledge such as implicit inference process; grounds of reasonings.

The dissertation suggests the teaching process to help students can conjecture including basic steps: observation; using intellectual activities such as: analysis, summarizing, comparison, generalization; and inferences: similar, inductive; mathematical extrapolation and demonstration to recognize and predict new object or predict solutions for new problem; using mathematical inferences to confirm or reject the prediction.

Besides, the dissertation also suggests process for students to make sound reasonings including some basic steps: learners analyze logical structure of the problem; the teacher organizes students to analyze solutions for the problem; Students evaluate grounds of each step in reasoning of problem solving or define the right premises using in problem solving; Students present the answer for the problem.

The author has proceeded pedagogic practice in two rounds at four schools: Ha Dong High School, Tran Hung Dao High School, Quang Trung High School and Tay Ho High School in Hanoi city. The results of the practice show the possibility and effectiveness of suggested methods.